

Recovering Entanglement by Local Operations

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We show that any quantification of the bipartite entanglement of mixed states uniquely based on the density operator may lead to a paradoxical increase of entanglement under purely local operations. This apparent paradox is solved in the physical ensemble description of the system state by introducing the concept of "hidden" entanglement, which measures the amount of entanglement that may be recovered without the help of any *non-local* operation. For two noninteracting qubits under a low-frequency stochastic noise, we show that entanglement can be recovered by local pulses only. We also discuss how hidden entanglement may provide new insights about entanglement revivals in non-Markovian dynamics. We finally propose a simple quantum information scheme, implementable by all-optical setups, which gives evidence of the concept of hidden entanglement.

PACS numbers: 03.67.-a, 03.65.Ud, 03.65.Yz

Introduction.—Entanglement, arguably the most peculiar feature of quantum mechanics, plays a key role in several quantum information and communication applications, including teleportation, quantum dense coding, private key distribution, and reduction of communication complexity [1–4]. To work properly, all the above tasks generally require pure maximally entangled states. Since entanglement cannot be generated by Local Operations and Classical Communication (LOCC), entangled states must be generated somewhere, and then they have to be distributed among different parties, possibly far away from each other (*transmission*) [3, 4]. Once entanglement has been distributed, it can then be used immediately or stored, waiting for the time to use it (*storage*). Systems which are physical supports for entangled states, unavoidably interact with the environment, both during transmission and storage times, and therefore undergo noisy processes that damage entanglement. For practical purposes it is necessary to quantify the amount of entanglement that a system loses during time evolution.

For a pure state $\rho = |\psi\rangle\langle\psi|$, bipartite entanglement between subsystems A and B is unambiguously defined as the *entropy of entanglement* $E(|\psi\rangle\langle\psi|) = S(\rho_A) = S(\rho_B)$, where $S(\rho_i)$ is the von Neumann entropy of one of the two reduced states, $\rho_A = \text{Tr}_B \rho$ and $\rho_B = \text{Tr}_A \rho$. The quantification of entanglement for mixed states is a much more complicated, open problem [3, 4]. The difficulty here is rooted in the fact that a mixed state ρ may be decomposed into an ensemble of pure states in an infinite number of ways, $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, with $p_i > 0$ and $\sum_i p_i = 1$. The arbitrariness of the decomposition renders any quantification of mixed-state entanglement difficult, since some optimization over all possible decom-

positions is needed.

In this Letter, we show that any quantification of entanglement uniquely based on the density operator ρ may lead to the paradoxical result that the entanglement of bipartite systems can increase under local operations. Since local operations cannot create entanglement, the increase of entanglement must be attributed to the manifestation of quantum correlations that were already present before the application of the local operation itself. These quantum correlations are in some sense *hidden*, since the density operator formalism does not capture their presence. We propose a quantitative definition of Hidden Entanglement (HE) and illustrate the usefulness of this concept by several examples. We show that the presence of HE allows to recover entanglement between two noninteracting qubits subject to a low-frequency stochastic environment by *local* pulses (acting only on one qubit). We also discuss the relation between HE and the occurrence of entanglement revivals, that is an up-to-date open issue. Finally, we propose a quantum information scheme where the presence of HE can be revealed.

Hidden entanglement.—We deal with an ensemble $\mathcal{A} = \{(p_i, |\psi_i\rangle)\}$ of systems, that is, we know the statistical distribution of the bipartite pure states $\{|\psi_i\rangle\}$, occurring with probabilities $\{p_i\}$, so that $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, but we do not know the state of any individual system in the ensemble. The amount of entanglement present in the ensemble \mathcal{A} can be properly identified as its *average entanglement* [5–9]:

$$E_{av}(\mathcal{A}) = \sum_i p_i E(|\psi_i\rangle\langle\psi_i|). \quad (1)$$

If each system in the ensemble evolves during time t under LOCC, the maximum amount of entanglement of the corresponding density operator $\rho(t)$ can never overcome the initial value $E_{av}(\mathcal{A})$. This statement can be proved by the following simple argument. Suppose Charlie prepares a bipartite system in a (possibly entangled) pure state of the ensemble \mathcal{A} . Then he sends one half of the system to Alice and the other half to Bob through noiseless quantum channels. Alice and Bob are linked only by a noiseless classical channel. Charlie repeats this operation N times. Among these, a certain number M_j of times Alice and Bob deal with the state $|\psi_j\rangle$. If Alice (or Bob) receives from Charlie the classical information about which state he sent each time, in the limit of large N Alice and Bob can distill - by only using LOCC - up to $M_j E(|\psi_j\rangle\langle\psi_j|)$ maximally entangled states from the M_j states $|\psi_j\rangle$ at their disposal [5]. Distillable entanglement is in fact the entropy of entanglement for pure bipartite states. Therefore, the maximum entanglement that Alice and Bob can distill per pair, by using the classical information received from Charlie, is

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_i M_i E(|\psi_i\rangle\langle\psi_i|) = \sum_i p_i E(|\psi_i\rangle\langle\psi_i|), \quad (2)$$

where $\lim_{N \rightarrow \infty} \frac{M_i}{N} = p_i$, that is just the average entanglement of Eq. (1).

We define the *hidden entanglement* of the ensemble $\mathcal{A} = \{(p_i, |\psi_i\rangle)\}$ as the difference between the average entanglement and the entanglement [3, 4] of the state $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ quantified by any convex measurement $E(\rho)$ (reducing to the entropy of entanglement for pure states), that is

$$E_h(\mathcal{A}) \equiv E_{av}(\mathcal{A}) - E(\rho) = \sum_i p_i E(|\psi_i\rangle\langle\psi_i|) - E\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right). \quad (3)$$

Thanks to convexity, E_h is always larger than or equal to zero. The meaning of the hidden entanglement of Eq. (3) can be understood as follows: it measures the entanglement that cannot be exploited as a resource due to the *lack of knowledge* about which state of the mixture we are dealing with. Such entanglement can be recovered (unlocked) once this (classical) information is provided [10]. There exist several inequivalent measures of mixed state entanglement [3, 4]. Here we consider the entanglement of formation $E_f(\rho)$, which is an upper bound for any bipartite entanglement measure [13], so that $E_{av} - E_f(\rho)$ is a lower bound for the hidden entanglement. $E_f(\rho)$ can be readily computed for two-qubit systems via the concurrence $C(\rho)$ [14].

Random local fields.—We first illustrate the concept of HE by considering a random, local dynamics and demonstrating that, under proper conditions, a *complete recovery* of the entanglement $E_f(\rho)$ of the state

may occur. A basic property of the average entanglement is its invariance under local unitary transformations acting on any system state $|\psi_i(t_0)\rangle$ in the ensemble $\mathcal{A}(t_0) = \{(p_i, |\psi_i(t_0)\rangle)\}$. In particular this is the case of evolution in a random local external field [15] inducing *local random unitaries* $U_\alpha(t) \otimes V_\beta(t)$, with the operators U_α and V_β acting on the first and on the second subsystem, respectively, and α, β random variables. The average entanglement is conserved by this dynamics, $E_{av}(\mathcal{A}(t)) = E_{av}(\mathcal{A}(t_0))$. On the other side, the entanglement of the mixture is only upper bounded by the average entanglement: $E_f(\rho(t)) \leq E_{av}(\mathcal{A}(t))$, implying that a variable (time dependent) HE may exist. This is clearly illustrated by the following simple example.

Let us consider a two-qubit system AB initially prepared in the maximally entangled Bell state $|\phi^+\rangle$ (we denote the states of the Bell basis as $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$). The time evolution consists of local unitaries, but we have no complete information about which local unitary is acting. In particular, we suppose that the qubit A undergoes, with equal probability, a rotation about the x -axis of its Bloch sphere, $U_x(t) = e^{-i\sigma_x \omega t/2}$, or a rotation around the z -axis, $U_z(t) = e^{-i\sigma_z \omega t/2}$, while the qubit B remains unchanged. Hence, the ensemble \mathcal{A} at time t is $\mathcal{A}(t) = \{(\frac{1}{2}, (U_x(t) \otimes \mathbb{1}_B)|\phi^+\rangle), (\frac{1}{2}, (U_z(t) \otimes \mathbb{1}_B)|\phi^+\rangle)\}$. Since we are dealing with random local unitaries, the average entanglement is constant in time, $E_{av}(\mathcal{A}(t)) = 1$. On the other hand, the entanglement of the state $\rho(t)$ changes in time. At $\bar{t} = \frac{\pi}{\omega}$, $\rho(\bar{t}) = \frac{1}{2}|\phi^-\rangle\langle\phi^-| + \frac{1}{2}|\psi^+\rangle\langle\psi^+|$ is separable, whereas at $2\bar{t}$, $U_x(2\bar{t}) = U_z(2\bar{t}) = \mathbb{1}_A$ and the initial maximally entangled state is recovered. In the interval $[\bar{t}, 2\bar{t}]$ the entanglement revives from zero to one without the action of any nonlocal quantum operation, thus apparently violating the monotonicity axiom. The ensemble description tells us that at time \bar{t} the system is always in an entangled state ($|\phi_-\rangle$ or $|\psi_+\rangle$), but the lack of knowledge about which local operation the system underwent prevents us from distilling any entanglement: entanglement is *hidden*, $E_h(\mathcal{A}(\bar{t})) = 1$ and $E_f(\rho(\bar{t})) = 0$. At time $2\bar{t}$ this lack of knowledge is irrelevant since the two possible time evolutions result in the identity operation $\mathbb{1}_A$ and entanglement is recovered, $E_h(\mathcal{A}(2\bar{t})) = 0$ and $E_f(\rho(2\bar{t})) = 1$. We notice that entanglement revivals have already been observed for a two-qubit system under classical environments [16, 17] and they have been interpreted in terms of correlations in a classical-quantum state of environments and qubits [17], but not in terms of HE.

Stochastic low-frequency noise.—A fingerprint of the existence of HE is the possibility to completely recover entanglement of a noisy bipartite system by the action of *local* pulses. Here we consider a simple system consisting of two noninteracting qubits affected by stochastic low-frequency noise. This simplified model captures es-

sential features of several nanodevices whose dynamics is dominated by low-frequency noise [18]. We suppose the two qubits are initially prepared in a Bell state $|\varphi_0\rangle$ and, for the sake of simplicity, assume that only one of the two qubits, namely qubit A , is affected by phase noise (pure dephasing), as described by ($\hbar = 1$)

$$\mathcal{H}_A(t) = [-\Omega_A \sigma_z + \varepsilon(t) \sigma_z + \mathcal{V}(t, \bar{t}) \sigma_x]/2, \quad (4)$$

whereas qubit B evolves unitarily under Hamiltonian $\mathcal{H}_B(t)$. To start with, we suppose that the stochastic process $\varepsilon(t)$ is sufficiently slow to be considered static during the evolution time t , with a value randomly fluctuating from one quantum evolution (quantum trajectory [19]) to the other: ε is thus assumed to be a Gaussian random variable with zero expectation value and standard deviation σ . $\mathcal{V}(t, \bar{t})$ indicates an echo π pulse applied at time \bar{t} and assumed to be short enough to neglect the effect of noise during its application. The evolution operator during the pulse is $e^{-i\sigma_x \pi/2} = -i\sigma_x$. Static noise [18] produces an effect analogous to inhomogeneous broadening in Nuclear Magnetic Resonance (NMR) [20]. The system state is described by the quantum ensemble $\mathcal{A}(t) = \{p(\varepsilon)d\varepsilon, |\varphi_\varepsilon(t)\rangle\}$, where $|\varphi_\varepsilon(t)\rangle = \hat{T}e^{-i\int_0^t \mathcal{H}_A(t')dt'} \otimes \hat{T}e^{-i\int_0^t \mathcal{H}_B(t')dt'} |\varphi_0\rangle$. Note that during each trajectory the system acquires a random phase. In the density operator description of the system, the information about the random phase acquired by the system is disregarded by averaging the evolved pure state $|\varphi_\varepsilon(t)\rangle$ with respect to the random variable ε : $\rho(t) = \int d\varepsilon p(\varepsilon) |\varphi_\varepsilon(t)\rangle \langle \varphi_\varepsilon(t)|$. Here $p(\varepsilon)$ is the Gaussian probability density function of ε . Following Ref. [21], we can show that a system prepared in a Bell state evolves in a mixture $\rho(t)$ whose concurrence is

$$C(\rho(t)) = \begin{cases} e^{-\frac{1}{2}\sigma^2 t^2}, & 0 \leq t \leq \bar{t}, \\ e^{-\frac{1}{2}\sigma^2 (t-2\bar{t})^2}, & \bar{t} \leq t \leq 2\bar{t}. \end{cases} \quad (5)$$

The average entanglement of the evolved physical ensemble is $E_{av}(\mathcal{A}(t)) = 1$: during each trajectory, the system state remains in a pure maximally entangled state. Instead, the entanglement of formation $E_f(\rho(t))$ is obtained directly by $C(\rho(t))$ [14] and it is plotted in Fig. 1. In absence of pulses, E_f decays and almost vanishes ($E_f(\rho(t)) \simeq 0$) at times $\sigma t \gg 1$ (red curve). Differently, when a local echo pulse is applied at $t = \bar{t}$ the entanglement subsequently increases (blue curve), reaching at $t = 2\bar{t}$ its initial maximum value $E_f(\rho(2\bar{t})) = E_f^{\max} = 1$, which equals the average entanglement (dashed line). The entanglement of formation can also be evaluated when the stochastic process $\varepsilon(t)$ has a finite correlation time τ entering the autocorrelation function $\langle \varepsilon(t)\varepsilon(0) \rangle = \sigma^2 e^{-|t|/\tau}$. In this case, significant entanglement recovery is still achievable by a local echo pulse, provided that the noise correlation time is sufficiently longer than the time when the pulse is applied, $\tau \gg \bar{t}$ (dotted lines in Fig. 1).

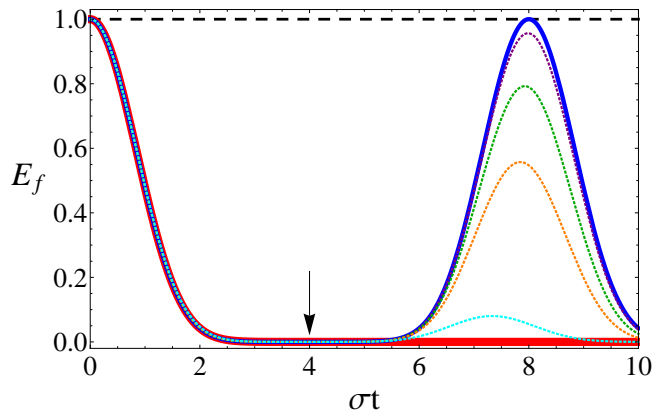


FIG. 1. (Color online) Entanglement of formation $E_f(\rho(t))$ as a function of the dimensionless time σt . The lower (red) solid curve corresponds to the free evolution in the presence of static noise, the upper (blue) solid curve is the result of the echo pulse applied at time $\sigma \bar{t} = 4$ (indicated by the arrow), from Eq. (5). The dashed line at the top is the system average entanglement $E_{av}(\mathcal{A}(t)) = 1$. Dotted curves are the numerical $E_f(\rho(t))$ evaluated for a stochastic process $\varepsilon(t)$ with correlation time $\sigma \tau$ and with the echo pulse applied. From bottom to top: $\sigma \tau = 20$ (cyan curve), 100 (orange curve), 200 (green curve), and 500 (purple curve). Perfect recovery is obtained in the limit $\tau/\bar{t} \rightarrow \infty$, corresponding to static noise (blue solid curve).

Therefore, the non-Markovian character of noise is crucial in order to uncover entanglement by local pulses.

This example shows the possibility to *fully recover the entanglement $E(\rho)$ by a local operation*. The physical mechanism behind this phenomenon is very simple: the local π -pulse refocuses the different qubit quantum trajectories restoring at time $2\bar{t}$ the qubit A coherence and consequently (qubit B evolves unitarily) causing the entanglement to reappear, with an efficiency depending on the correlation time of the stochastic process. Thus, entanglement can be recovered without any nonlocal control. At first sight this result may appear paradoxical, since entanglement is by definition a nonlocal resource. In fact, here entanglement is not destroyed during the time evolution, as indicated by the average entanglement which is maximum at any time, $E_{av}(\mathcal{A}(t)) = 1$. Entanglement is instead *hidden*, E_h increases due to defocusing among the different evolutions of the (maximally entangled) states of $\mathcal{A}(t)$.

From the examples above we argue that a complete recovery of entanglement is possible when various members of the physical ensemble evolve differently from each other but unitarily. In this case, even though the evolution of the ensemble averaged density matrix is not unitary, no qubit-environment entanglement is generated and environment can be described as a *classical system* [17]. Using a terminology borrowed from NMR [22] we speak of *incoherent errors*, whereas *decoherent errors* arise when the evolution is non-unitary even for a single member of the ensemble. For this latter case the average

entanglement decays.

We point out that HE depends on the quantum ensemble *physically* giving the system state. The key point is that the physical dynamics constrains the evolved density matrix of the system to a specific decomposition. In the above examples, the physical decomposition of the evolved density matrix is always an ensemble of maximally entangled states. Had we considered a dynamics such as to give, at $t = \bar{t}$, a mixture of separable states as decomposition of the system density matrix, no local operation would have been capable to recover entanglement ($E_h(\bar{t}) = 0$). The dynamics of the system after the application of the pulse proves that, at time \bar{t} , the two decompositions are not equivalent.

Finally, we remark that the results of the previous examples do not violate the monotonicity axiom [3–5]: *entanglement cannot increase under LOCC*. The key point is that this axiom is fulfilled by all entanglement measures provided we consider local operations which are Completely Positive, Trace preserving (CPT) maps. Conversely, in the previous examples entanglement recovery, from time \bar{t} to time $2\bar{t}$, is induced by purely local operations which are not LOCC, since the corresponding density matrix evolution cannot be described by a CPT map. To prove this point, it is enough to observe that the density matrix $\rho(2\bar{t}) = \rho(0)$, and therefore driving the system from the state $\rho(t)$ to $\rho(2\bar{t})$ is equivalent to driving it to $\rho(0)$. This operation cannot be described by a CPT map since the (CPT) evolution from time 0 to time \bar{t} is not invertible.

Discussion.—Now we want to show as the phenomenon of entanglement revivals that we have examined is conceptually different from the revivals that a system can exhibit due to the interaction with a quantum environment. Let us consider a very simple example, a two-qubit system A - B where A resonantly interacts with a quantum harmonic oscillator O via a Jaynes-Cummings (JC) Hamiltonian [1, 2], while B is isolated. Initially, the two qubits are prepared in the Bell state $|\phi_{AB}^+\rangle$ and O in its ground state $|0_O\rangle$. In the interaction picture, the Hamiltonian is $\mathcal{H}_{AO} = g(\sigma_+ a + \sigma_- a^\dagger)$, where g is the coupling constant, σ_+ (σ_-) the qubit rising (lowering) operator, and a^\dagger (a) the oscillator creation (annihilation) operator. At time $\bar{t} = \pi/g$ the states of A and O are swapped with respect the initial state, and the global state becomes $|0_A\rangle \otimes |\phi_{BO}^+\rangle$, giving $\rho_{AB} = |0_A\rangle \otimes \frac{1}{2}\mathbb{1}_B$: at time \bar{t} the A - B entanglement is zero, being completely transferred to B - O . Any unravelling of the A - B dynamics gives at this time a quantum ensemble whose average entanglement is zero, so that $E_h(\bar{t}) = 0$: local operations and classical communication cannot recover any entanglement between A and B . Only the subsequent interaction between A and O can gradually restore the A - B entanglement thanks to back-action (at time $2\bar{t}$, when a new A - O swapping is completed, the initial state is just retrieved). Therefore, the entanglement revival is here

due to the perfect entanglement back-transfer (a nonlocal operation), as well-known in the literature [21, 23, 24].

This explanation is unsuitable for the examples considered above in this paper, where the environment is classical and no entanglement transfer is possible. During the dynamics, the environment acquires only classical information about the system A - B and does not entangle with A or B . Quantum correlations do not leave the system A - B but they are simply not accessible due to the lack of classical information. Indeed, if at time \bar{t} ($E_h(\bar{t}) = 1$) someone provides A - B with the classical information about which random unitary the system underwent (in the case of random local fields) or about which random phase is added to the system states (in the case of stochastic pure-dephasing noise), then all the A - B entanglement could be recovered.

Often, the non monotonous behaviour of the entanglement between independent subsystems is assumed to be a signature a non-Markovianity [25]. However non-Markovian dynamics themselves cannot explain the occurrence of entanglement revivals between independent subsystems, since entanglement is a nonlocal resource. Non-Markovianity can be rather considered a necessary feature for such a dynamical behaviour, as one can see from the example we proposed about two qubits affected by low frequency noise. It may be fundamental to distinguish the origin of entanglement revivals depending on the nature of the environment. Entanglement can be (i) transferred forth and back due to some non-local interaction (quantum environments), or (ii) hidden and recovered by local operations (classical environments). From an operational point of view, the difference is significant: in case (i), when entanglement disappears, no local operation or classical communication may be helpful to restore it; in case (ii) we can act locally to recover entanglement.

Non-Markovianity has also been recently interpreted also as a “flow of information from the environment back to the system” [26]. A distinctive feature of the examples we have treated (*random local fields* and *stochastic low-frequency noise*) is that the information is here purely classical.

Quantum information scheme.—We now show an example implementable by current technologies in all-optical setups [27], where the presence of hidden entanglement can be detected by sharing private classical information. We assume that Alice prepares Bell’s pairs $|\phi^\pm\rangle$, and sends one half of each pair to Bob. Eve disturbs the communication with a phase shifter applied half of the times, in such a way that Alice and Bob eventually share either of the two states $|\phi^+\rangle$ and $|\phi^-\rangle$ with equal probabilities $p^+ = p^- = \frac{1}{2}$. Thus, the overall density matrix of Alice, Bob, and Eve reads

$$\rho_{ABE} = \frac{1}{2}|\phi^+\rangle\langle\phi^+| \otimes |\kappa^+\rangle\langle\kappa^+| + \frac{1}{2}|\phi^-\rangle\langle\phi^-| \otimes |\kappa^-\rangle\langle\kappa^-|, \quad (6)$$

where $\{|\kappa^+\rangle, |\kappa^-\rangle\}$ are two orthonormal states for Eve’s

system. The reduced density matrix of Alice and Bob, $\rho_{AB} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$, is separable. Nevertheless, if Alice and Bob agree to measure their qubits in the computational basis $\{|0\rangle, |1\rangle\}$, at the end of the transmission of each qubit they share a classical secret bit. One can prove this statement by calculating the correlations among the systems involved in the communication. We measure the correlations between two quantum systems Q_1 and Q_2 by the *quantum mutual information* [1, 2, 28] $I(Q_1 : Q_2) = S(\rho_{Q_1}) + S(\rho_{Q_2}) - S(\rho_{Q_1 Q_2})$. We then obtain that Eve knows which state Alice and Bob share ($I(AB : E) = 1$). However, Alice and Bob share a bit of classical information ($I(A : B) = 1$) and Eve has no knowledge about such bit ($I(A : E) = I(B : E) = 0$). This secret communication is possible due to the presence of hidden entanglement. In fact, Alice and Bob share entangled pairs with a statistical distribution described by the ensemble $\mathcal{A} = \{(1/2, |\phi^+\rangle), (1/2, |\phi^-\rangle)\}$, having $E_{av}(\mathcal{A}) = E_h(\mathcal{A}) = 1$. Notice that, if Alice and Bob share separable two-qubit states taken from some ensemble \mathcal{B} , we have $E_h(\mathcal{B}) = 0$ and it is straightforward to show that, each time, Eve knows which state they are sharing: private classical communication is not possible in this case.

Conclusions.— We have introduced the concept of hidden entanglement based on the quantum trajectory description of the system dynamics, and consequently on the quantum ensemble description of the system state. We have shown that a recovery of entanglement may be performed by only local operations, provided that hidden entanglement (HE) is present. There is no violation of the entanglement monotonicity axiom because no entanglement is destroyed and created in this case: a nonzero HE signals a loss of entanglement that is not due to the establishment of quantum correlations with the environment. Quantum correlations remain within the system, but they are not exploitable due to the lack of classical information.

We have shown that HE can be of practical relevance in solid-state devices which prevalently suffer from low-frequency noise, indicating the amount of entanglement that can be recovered by standard local pulses, independently of how far apart the qubits are. This allows us, differently from other proposals [29], to avoid resorting to non-local gates which may be a much more demanding task. The concept of HE can be also applied in the case of environment monitoring strategies [30, 31]. In these cases, however, the environment cannot be described as a classical system and therefore information cannot be retrieved by only local operations: “environment measurements” are also needed to unlock HE. We have finally proposed a quantum information scheme where the presence of HE can be detected by sharing of private classical information.

R.L.F. thanks Giuseppe Compagno for useful discussions. This work was partially supported by the EU

through grant no. PITN-GA-2009-234970, and by the Joint Italian-Japanese Laboratory on “Quantum Technologies: Information, Communication and Computation” of the Italian Ministry of Foreign Affairs.

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